

# NUCLEON FORM FACTORS FROM DISPERSION THEORY

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I review the results of a recent dispersion-theoretical analysis of the nucleon electromagnetic form factors and comment on the strangeness form factors. The need for a better data basis at low, intermediate and large momentum transfer is stressed.

## 1 WHY DISPERSION RELATIONS ?

The structure of the nucleon (denoted by 'N') as probed with virtual photons is parametrized in terms of four form factors,

$$\langle N(p') | \mathcal{J}_\mu | N(p) \rangle = e \bar{u}(p') \left\{ \gamma_\mu F_1^{p,n}(t) + \frac{i\sigma_{\mu\nu} k^\nu}{2m_N} F_2^{p,n}(t) \right\} u(p) , \quad (1)$$

with  $t = k_\mu k^\mu = (p' - p)^2$  the invariant momentum transfer squared,  $\mathcal{J}_\mu$  the em current related to the photon field and  $m_N$  the nucleon mass. In electron scattering,  $t < 0$  and it is thus convenient to define the positive quantity  $Q^2 = -t > 0$ .  $F_1$  and  $F_2$  are called the Pauli and the Dirac form factor (ff), respectively. There exists already a large body of data for the proton and also for the neutron. In the latter case, one has to perform some model-dependent extractions to go from the deuteron to the neutron. More accurate data are soon coming (ELSA, MAMI, CEBAF, ...). It is thus mandatory to have a method which allows to analyse all these data in a mostly model-independent fashion. That's where dispersion theory comes into play. Although not proven strictly (but shown to hold in all orders in perturbation theory), one writes down an unsubtracted dispersion relation for  $F(t)$  (which is a generic symbol for any one of the four ff's),

$$F(t) = \frac{1}{\pi} \int_{t_0}^{\infty} dt' \frac{\text{Im } F(t')}{t' - t} , \quad (2)$$

with  $t_0$  the two (three) pion threshold for the isovector (isoscalar) ffs.  $\text{Im } F(t)$  is called the spectral function. These spectral functions are the natural meeting ground for theory and experiment, like e.g. the partial wave amplitudes in  $\pi N$  scattering. If the data were to be infinitely precise, the continuation from negative  $t$  (data) to positive  $t$  (spectral functions) in the complex- $t$  plane would lead to a unique result for the spectral functions. Since that is not the case, one has to make some extra assumption guided by physics to overcome the ensuing instability as will be discussed below. Let me first enumerate the various constraints one has for the spectral functions.

## 2 CONSTRAINING THE SPECTRAL FUNCTIONS

In general, the spectral functions can be thought of as a superposition of vector meson poles and some continua, related to n-particle thresholds, like e.g.  $2\pi$ ,  $3\pi$ ,  $K\bar{K}$  and so on. For example, in the Vector Meson Dominance picture one simply retains a set of poles. However, there are some powerful constraints which these spectral functions have to obey:

- *Unitarity*: As pointed out by Frazer and Fulco [1] long time ago, extended unitarity leads to a drastic enhancement of the isovector spectral functions on the left-wing of the  $\rho$  resonance. Leaving out this contribution from the two-pion cut leads to a gross underestimation of the isovector charge and magnetic radii. This very fundamental constraint is very often overlooked. It is believed that in the three pion (isoscalar) channels no such enhancement exists.
- *pQCD*: Perturbative QCD (pQCD) tells us how the nucleon ffs behave at very large momentum transfer based on dimensional counting arguments supplemented with the leading logs due to QCD [2]. This leads to a set of superconvergence relations for  $\text{Im } F_1(t)$ ,  $\text{Im } F_2(t)$  and  $\text{Im } t F_2(t)$ , which have to be imposed ( $F_2$  is suppressed by one more power in  $t$  than  $F_1$  due to the spin-flip).
- *The neutron radius*: Over the last years, the charge radius of the neutron has been determined very accurately by measuring the neutron-atom scattering length, i.e.  $F_1^n$  at  $Q^2 = 0$ . This value, which we take from the recent paper [3], has to be imposed as a further constraint.
- *Stability*: The isovector spectral functions are completely fixed from  $t = (4 \dots 50) M_\pi^2$  due to unitarity. At large  $t$ , pQCD determines the behaviour of all isovector/isoscalar spectral functions. In addition, we have a few more isovector and isoscalar poles. Loosely spoken, their number is minimized by the requirement that the data can be well fitted (for details, see Refs.[4,5]).

### 3 RESULTS AND DISCUSSION

The spectral functions fulfilling all the abovementioned requirements consist of a hadronic part (the  $2\pi$  continuum plus three additional isovector and isoscalar poles) and the quark contribution leading to the pQCD behaviour (parametrized by a simple log function which depends on a parameter  $\Lambda^2$  that can be considered a measure of separating these two regimes) [4]. At this point, we have 8 free parameters. In contrast to a previous dispersive analysis [5], we are able to identify all three isoscalar and two of the isovector masses with physical ones. Only the third isovector mass is so tightly fixed by the constraints that it can not be chosen freely. This leaves us with effectively three fit parameters. The best fit to the nucleon form factors is shown in Fig.1 (to be precise, we show the ffs normalized to the dipole fit, in case of  $G_E^n$  we normalize to the Platchkov data [6] for the Paris potential). From these, we deduce the following nucleon electric (E) and magnetic (M) radii [4]:

$$r_E^p = 0.847 \text{ fm}, \quad r_M^p = 0.836 \text{ fm}, \quad r_M^n = 0.889 \text{ fm}, \quad (3)$$

all with an uncertainty of about 1%. These results are similar to the ones found by Höhler et al. [5] with the exception of  $r_M^n$  which has increased by 5% (due to the neglect of one superconvergence relation in Ref.[5]). From the residues at the two lowest isovector poles, we can determine the  $\omega NN$  and  $\phi NN$  coupling constants,

$$\frac{g_{\omega NN}^2}{4\pi} = 34.6 \pm 0.8, \quad \kappa_\omega = -0.16 \pm 0.01, \quad \frac{g_{\phi NN}^2}{4\pi} = 6.7 \pm 0.3, \quad \kappa_\omega = -0.22 \pm 0.01, \quad (4)$$

where  $\kappa_V$  ( $V = \omega, \phi$ ) is the tensor to vector coupling strength ratio. These results are similar to the ones in Ref.[5].

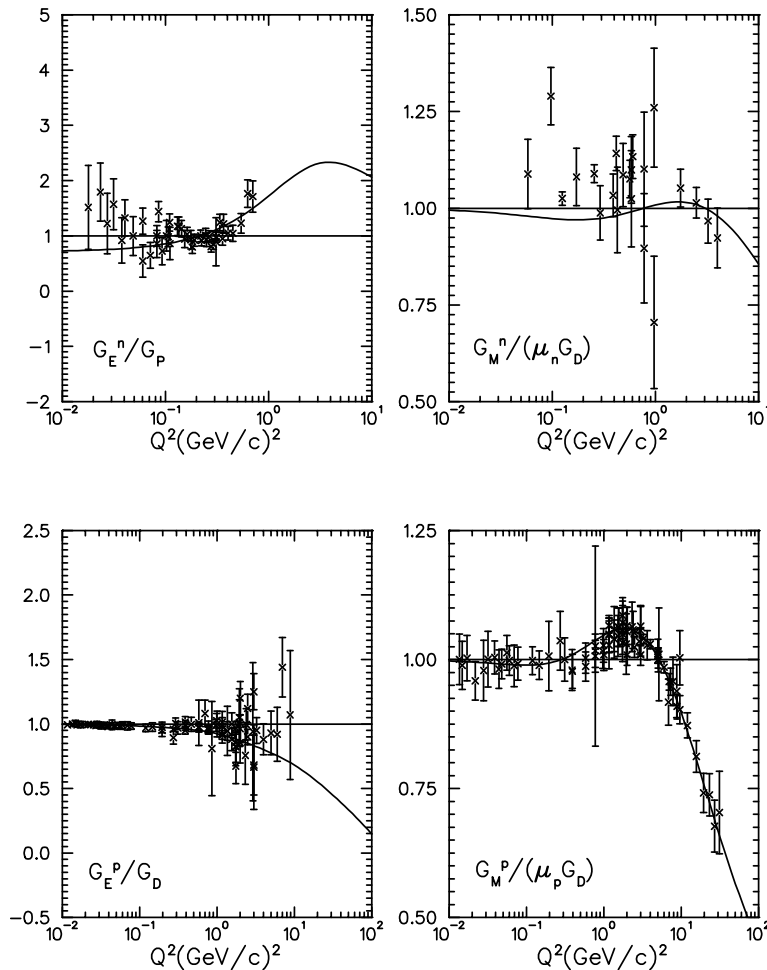


Fig. 1: Best fit to the nucleon em form factors

Of particular interest is the onset of pQCD. Only for  $G_M^p(t)$  data for  $Q^2 > 10 \text{ GeV}^2$  exist. While these data are consistent with the pQCD scaling  $L^{-1}(Q^2)Q^4 G_M^p(Q^2) \rightarrow \text{constant}$ , where  $L^{-1}(Q^2)$  accounts for the leading logs, they are not precise enough to rule out a non-scaling behaviour, see the lower panel in Fig.2. Also shown in Fig.2 is the same quantity without the log corrections (upper panel). All data for the much discussed quantity  $Q^2 F_2^p(Q^2)/F_1^p(Q^2)$  are below  $Q^2 = 10 \text{ GeV}^2$  which in our approach is still in the hadronic region since  $\Lambda^2 \simeq 10 \text{ GeV}^2$  for the best fit.

## 4 STRANGE FORM FACTORS

Jaffe [7] has shown how one can get bounds on the strange vector form factors in the nucleon from such dispersion theoretical results. The main assumption of his approach is that these strange form factors have the same large- $t$  behaviour as the non-strange isoscalar ones. If the fall-off for the strange form factors is faster, the strange matrix elements will be reduced. Using our best fit together with a better treatment of the symmetry breaking in the vector nonet, it is straightforward to update Jaffe's analysis. We find for the strange magnetic

moment and the strangeness radius [8],

$$\mu_s = -0.24 \pm 0.03 \text{ n.m.}, \quad r_s^2 = 0.21 \pm 0.03 \text{ fm}^2. \quad (5)$$

Furthermore, the strange ff  $F_2^s(t)$  follows a dipole with a cut-off mass of 1.46 GeV,  $F_2^s(t) = \mu_s/(1 - t/2.41 \text{ GeV}^2)^2$ . It is important to stress that these numbers should be considered as upper bounds.

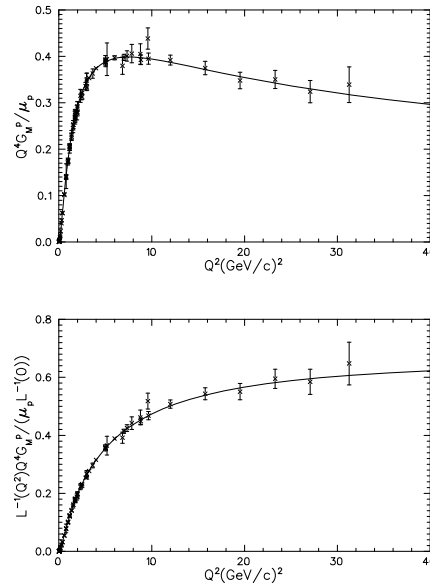


Fig. 2: pQCD scaling in  $G_M^p(Q^2)$ ? Upper/lower panel: Without/with leading logs.

## 5 SUMMARY

The dispersion theoretical machinery has been updated to include theoretical concepts like pQCD scaling and so on [4]. Now we need a more *accurate* data basis at *low, intermediate* and *large* momentum transfer to further sharpen the extractions of the nucleon em radii, the  $VNN$  coupling constants and to pin down the onset of perturbative QCD.

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